Post Lab 1: Resistors and Resistive Networks

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Consider the circuit shown in Figure 1. This is called an R-2R ladder network for fairly obvious reasons. Networks of this type can be used as digital-to-analog converters, as will be shown in this post lab exercise.

Upon first inspection of the circuit, we can perform a node-based analysis. We label nodes with the same indexing system as used for the currents shown in Figure 1 (e.g. node 1 is the node immediately above current I_1 in the diagram).

Starting at node 0, we find that the equivalent resistance $R_{eq,0}$ between this node and ground is simply

$$R_{eq,0} = 2R \| 2R = \frac{1}{\frac{1}{2R} + \frac{1}{2R}} = \frac{1}{\frac{2}{2R}} = R \tag{1}$$

Similarly, the equivalent resistance at node 1 is given as

$$R_{eq,1} = 2R \| (R + R_{eq,0}) = 2R \| 2R = R \tag{2}$$

We quickly realize that the equivalent resistance between any node k and ground is simply

$$R_{eq,k} = R \tag{3}$$

We can apply this knowledge as we conceive of each successive node's voltage as a simple division of the previous node's voltage. For node 0, we find that

$$V_0 = \frac{R_{eq,0}}{R + R_{eq,0}} V_1 = \frac{R}{R+R} V_1 = \frac{1}{2} V_1 \tag{4}$$

Similarly for node 1, we find its voltage is given as

$$V_1 = \frac{R_{eq,1}}{R + R_{eq,1}} V_2 = \frac{1}{2} V_2 \tag{5}$$

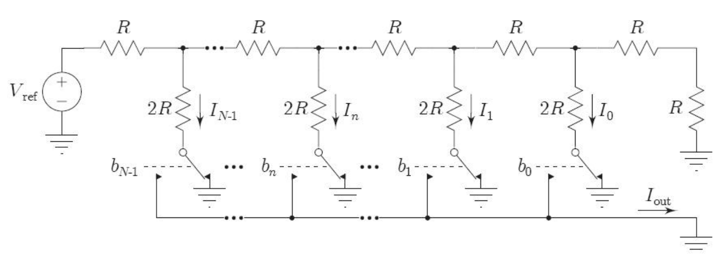

Figure 1: Diagram of an N step R-2R Ladder Network.

We quickly realize that the divider ratio between any two consecutive nodes is simply $k = \frac{1}{2}$. Thus, we may generalize the voltage at any arbitrary node n to be

$$V_n = \frac{1}{2} V_{n+1} \tag{6}$$

Transforming this definition from implicit to explicit form, we determine that in an N step ladder network like that in Figure 1 with source voltage V_{ref}, the voltage at some node n is given as

$$V_n = \frac{2^n}{2^N} V_{ref} \tag{7}$$

Applying Ohm's Law gives the current at node n as follows

$$I_n = \frac{V_n}{2R} = \frac{1}{2R} \frac{2^n}{2^N} V_{ref} = \frac{2^n}{2^{N+1}} \frac{V_{ref}}{R} \tag{8}$$

We know that each branch carries a current which is exactly twice that of its neighbor to the right. We can also compute the current at any node n as a function of the system's reference voltage V_{ref}, number of steps N, and unit resistance R. We then begin to understand that the outputs of all these branches, once scaled by the appropriate known constants, may collectively form a binary number system, with each branch contributing the amount allocated to a given "place value" . The current through branch 0, for example, contributes to the units place ($2^0 = 1$), while branch 2 contributes to the twos place ($2^1 = 2$).

Given this information, we may now consider the lower portion of Figure 1, in which we see binary switches directing the current through each branch either to ground (when $b_n = 0$) or to a aggregating branch with current I_{out} (when $b_n = 1$). This allows for each "place value" to be assign to either 0 (corresponding to ground) or 1 (corresponding to I_{out}), and lets all the nonzero binary place values become aggregated into one single value. Thus, the current I_{out}, when scaled properly, gives an analog value corresponding to a binary number defined by the digits b_{N-1} ... b_0. We have a digital-to-analog converter!

As a final step, finding the value of I_{out} explicitly is rather simple, as it is simply the sum of each individual branch current scaled by its corresponding binary digit. A mathematical expression is given below

$$I_{out} = \sum_{n=0}^{N-1} I_n b_n = \frac{V_{ref}}{2^{N+1} R} \sum_{n=0}^{N-1} 2^n b_n \tag{9}$$