Brian Fahrenbach and Michael Hughes
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Lab 4: Designing An Optimal Rectangular Plate Heat Sink

Abstract:

In this lab exercise, our team was tasked with designing an optimal heat sink using a block of aluminum. Our team chose to explore the space of multi-fin rectangular plate heat sinks and find an optimal design with analytical tools. Utilizing available resources heat sink optimization, we developed governing equations for our problem space and used computation software to model performance at various values of multiple geometric parameters to find the best design. We then had the design fabricated and experimentally tested. Our design proved to be the best of the nearly forty designs tested in terms of specific heat conductivity, validating our analytical approach.

Problem Specification:

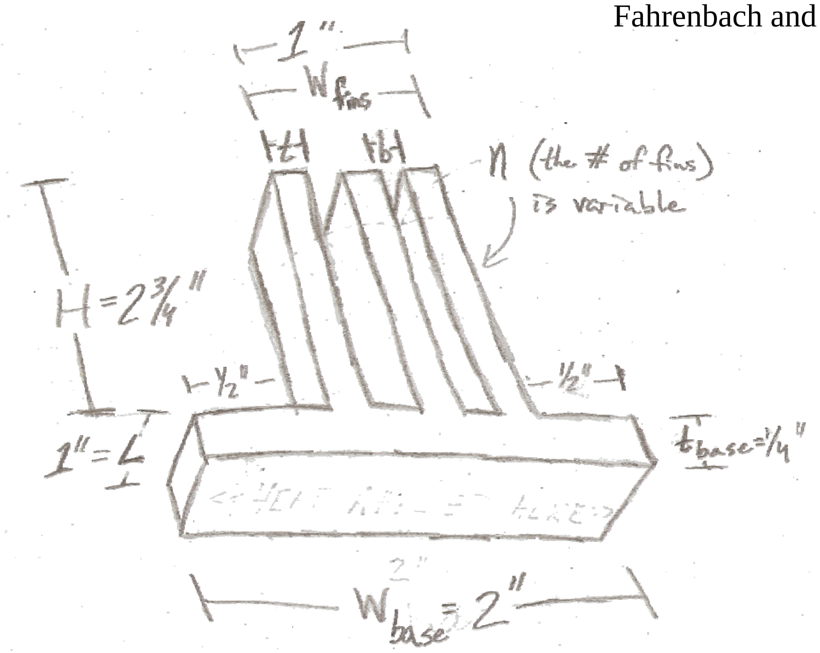
The problem specified calls for the design of a heat sink from a 3 by 3 by 1 inch which, when applied to a constant source temperature of 50 C in standard ambient air conditions, will dissipate the most heat for the least material cost. The fitness function of the heat sink is given below:

$$fitness = \frac{Q}{m\Delta T}$$

where Q is the heat dissipated (in Watts), deltaT is the temperature difference (in K) between the heat source and the ambient air, and m is the heat sink's mass (in kg). Another name for this fitness value is specific thermal conductivity, or STC. Our goal is to achieve a heat sink within the given constraints that yield the highest fitness value, since a design that does this will be optimal within our specifications.

Geometric Specifications:

In order to expedite the testing process for many heat sink designs, we were given certain constraints about the base dimensions. To comply with these constraints and maintain a simple, easy-to-model problem space, we chose to confine our design to the following multiple rectangular plate fin pattern:



Where the L is the fixed length of the block, Wbase is the fixed width of the block base, Wfins is the fixed total width of all fin space, H is the height of each fin, n is the number of fins, tbase is the mandated thickness of the base, t is the thickness of each fin, and b is the band gap width between each fin.

We were physically constrained in the following dimensions:

L = 1”, Wbase = 2”, Wfins = 1”, tbase = ¼”

Additionally, we made the choice to constrain H to its maximum possible value of 2 ¾ “, figured from the available dimensions of the block. We justify this choice with our qualitative understanding of convection. A large H value will maximize exposed surface area, which will in turn maximize the amount of heat dissipated, as described in Newton’s Law of Cooling.

The band gap width between each fin is dependent on other parameters as follows:

$$b = \frac{W_{fins} - n\,t}{n-1}$$

We thus have two remaining variables, *n* and *t*, and they will be the focus of our optimization.

Optimization Overview:

In order to find the geometry which would result in the optimal heat sink, we first needed to develop a system of governing equations which would allow us to accurately predict the convection and conduction occurring in a given geometry. After some preliminary online research, we found some promising methodologies, mostly in the work of J. Richard Culham at the University of Waterloo and Robert Simons of IBM.

Using these sources, we found expressions that allowed us to calculate the average convection coefficient of a heat sink and its thermal resistance when given the proper physical constants and the specific geometrical parameters described above. These equations will be presented and explained in more detail later. We then used general thermodynamics equations to determine analytically the total amount of heat dissipated by the heat sink. We could also calculate the mass of the heat sink using a calculation of volume and density. Thus, we were able to determine all three factors (heat dissipated, mass, and temperature difference) that were used in the fitness function of the heat sink.

With the ability to calculate the fitness of any given geometry, we could then run our calculations over a range of geometries, specifically different values of n and t, and determine the combination of those two variables which would result in optimal heat sink performance.

Calculation Details:

Calculating the convection coefficient for a particular heat sink geometry and temperature conditions proves to be a difficult endeavor. Luckily, we were able to find in the work of Simons [1] an empirically validated method for our particular geometry. The method, which utilizes physical constants, heat sink geometry, and dimensionless numbers is presented below:

We find the convection coefficient h with the equation

$$h = \frac{k_{air}\, Nus}{b}$$

Where $kair$ is air's thermal conductivity and Nus is the Nusselt number, defined as

$$Nus = \frac{1}{24}\, Ral \left(1 - e^{\frac{35}{Ral}}\right)^{3/4}$$

And Ral is the Rayleigh number, given by Culham et. al. [2] as

$$Ral = \frac{g\, \beta\, b^4\, \Theta}{\alpha\, v_{air}\, L}$$

where theta is temperature difference of the heat sink,

$$\Theta = T - T_{amb}$$

and the remaining physical constants are the gravitational constant, g, air's coefficient of thermal expansion *beta*, air's thermal diffusivity *alpha*, and air's kinematic viscosity, *vair*. Physical constant values were found using WPI's Air Property Calculator [3] as well as lookup tables presented by Wikipedia.

After calculating the convection coefficient, we can determine the thermal resistance of the heat sink. The work of Culham and Muzychka [4] shows that the total resistance of the sink, given our geometry, can be found in the expression:

$$Rsink = \frac{1}{\frac{n}{R_{fin}} + h\,(n-1)\,b\,L} + \frac{t_{base}}{k_{al}\,L\,W_{base}}$$

We note that the above expression makes qualitative sense, since it adds the resistances of fins and exposed base areas working in parallel. From our experience with electronics we know that a parallel network's total resistance is reciprocal of the sum of all the reciprocals of each element's resistance. Thus, the *n/Rfin* expression describes sum of the reciprocals of each fin resistance element, while the *h(n-1)bL* expression gives the sum of the reciprocals of the resistance of each exposed area of the base. We also included the conductivity resistance part of the expression (*tbase/(kal *L*Wbase))* for complete accuracy, though it will probably factor in very little since the thermal conductivity of aluminum is so great.

We further use the work of Culham and Muzychka to find the thermal resistance of each fin. He presents an empirically validated expression shown below

$$R_{fin} = \frac{1}{\sqrt{h\,P\,k\,A_c}\,\tanh(m\,H)}$$

Where *Ac* is the cross-sectional area (L*t) of each fin, P is the cross-sectional perimeter (2L + 2t), and *m* is defined as

$$m = \sqrt{\frac{h\,P}{k\,A_c}}$$

The mathematics here exceed the scope of our thermodynamics knowledge, so we plead ignorance on the details of this expression, confident in the knowledge that the work of Culham and others has refined it to sufficient accuracy.

With the ability to compute the total resistance of the heat sink, we are able to determine the total heat dissipated (Q) using the thermal equivalent of Ohm's Law:

$$Q = \frac{\Theta}{R_{sink}}$$

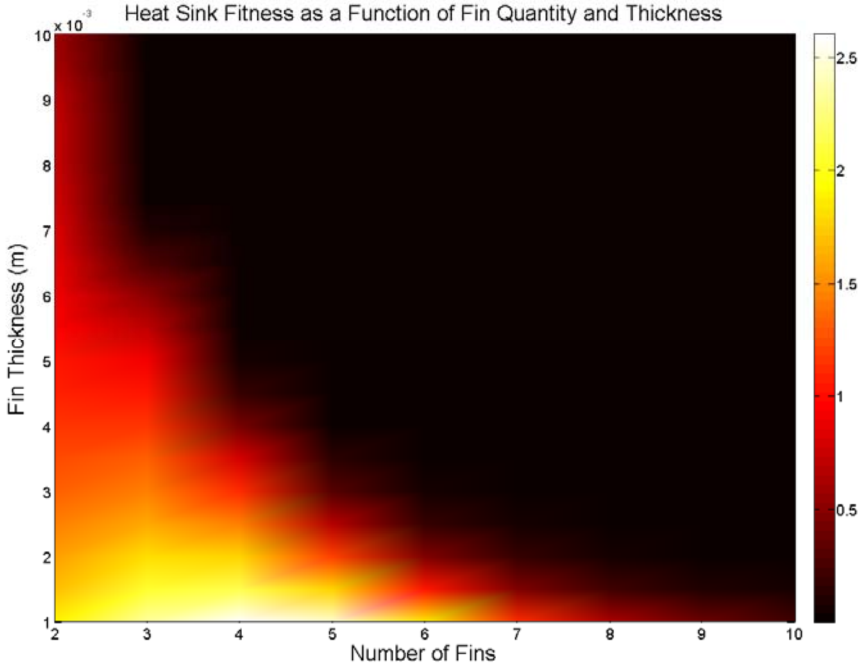
We can easily predict the mass of the heat sink using a volume and density calculation::

$$mass = \rho_{al}\,(t_{base}\,W_{base}\,L + n\,H\,L\,t)$$

Thus, we have the ability to calculate in detail the fitness of a proposed heat sink geometry.

Optimization Results:

We ran the calculations described above over a realistic range of values for *n* and *t,* and we obtained the following contour plot, displaying in color the fitness of a heat sink .



According to the figure, heat sinks in the context of our problem perform best when the fins are as thin as possible and few in number (between 2 and 6), which makes sense given that our fitness function favors those with low mass. The highest fitness occurs when the heat sink is made of up 4 fins, each 1 mm thick. At these dimensions, we calculate the following expected values:

h = 7.7719 W/m^2   Q =2.6854 W   mass = 41.3 g

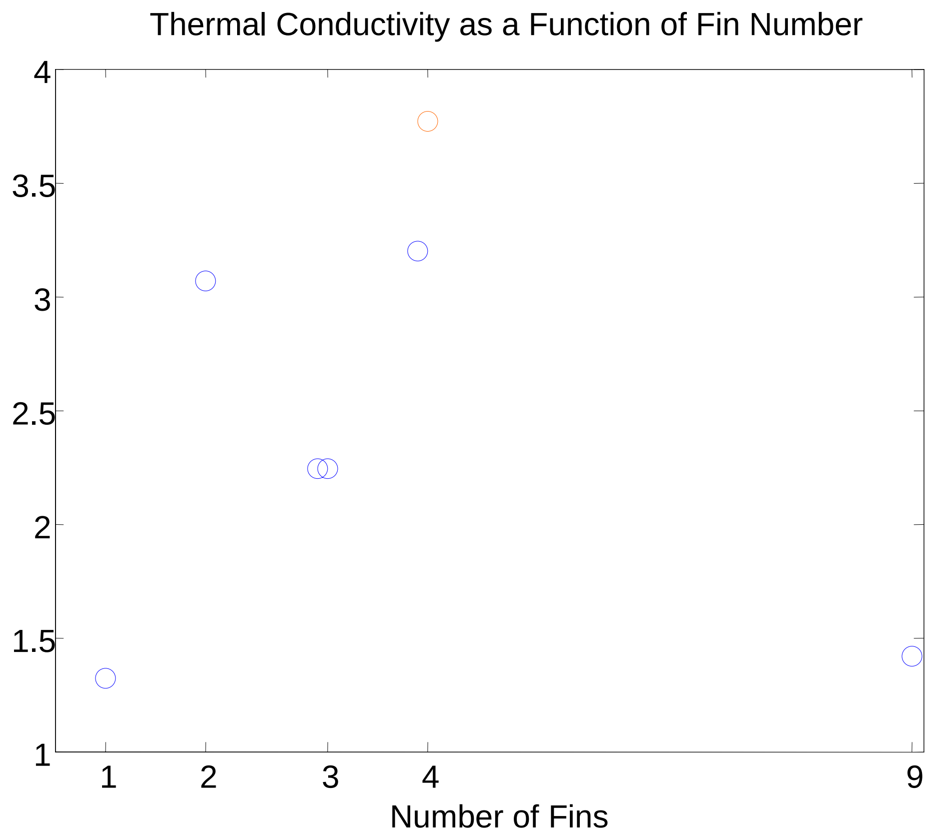
fitness = specific TC = 2.6019 W/(kg*K)

All of these seem to be realistic in the context of the problem, especially the convection coefficient, which was enough for us to be convinced in our model and move on the the fabrication and experimental testing stage.

Experimental Results:

After fabricating the heat sink using the water-jet cutter, we tested it using a procedure developed by our professor, Brian Storey.

At the end of all testing, it was found that our heat sink performed the best out of all the sinks submitted by members of our class.



Above we see a graph of the Thermal Conductivity as a function of the number of fins for the various heat sinks in class which used rectangular plate fins like ours did. While our fin (noted in red) has the highest TC, there is no linear correlation between the number of fins and the TC. It has more to do with optimizing the balance between surface area and mass. However, these experimental results may not be the best for graphing TC as a function of the number of fins, because the examples we have where 3 fins are used, the fins seem to be shorter than the fins in other heat sinks. However we can see that when you have a high number of fins, like 9, the ration of surface area to mass is no longer optimal, and the TC drops.

Specific observed property values for our heat sink are found below:

Q = 4.33 W   mass: 40 g   Specific TC: 3.77 W/(kg*K)

Our calculation ended up severely under-predicting the performance of our heat sink, as our predicted fitness of 2.6 W/(kg*K) differs from the actual measured value of 3.77 by 31%. Our mass calculation was fairly accurate (error of only 3.25%), so the discrepancy must have come from our computation of the sink's resistance. We did not take into account convection effects on the exposed "wings" of the heat sink base in this calculation, which probably accounts for the error. After all, these wings has a surface area of (Wbase – Wfins) * L = 1 square inch, a significant amount which would raise the amount of heat dissipated considerably. Additional error could result from a difference in ambient temperature values (our model used 25 C, while the experiment used 20.5 C).

Our computation of *h* and *Rfin* may not have been entirely exact, either, though it is beyond the scope of this work and our knowledge to determine the extent of this discrepancy with any certainty.

Despite these quantitative shortcomings, our model still accurately provided us with our ultimate qualitative goal: an optimized heat sink geometry. We can thus conclude that our analytical model, when considered against alternative methods like finite element analysis, provides a far more efficient and accurate prediction about the optimal performance of a rectangular plate heat sink.

Bibliography

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